



White paper by:  
US Math Recovery® Council

Math Recovery® Overview:  
An elementary school implementation of an early intervention program to  
identify and service of low-attaining mathematics students

Related program  
Add+VantageMR®: Efficient and effective assessment to support using data  
to make instructional decisions

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## ***Problem Statement***

The construction of numeracy skill, much like literacy skill, depends on certain well-defined habits of mind (counting, sequencing, grouping, and patterning) that directly correlate to a child's development. These mental habits can be addressed early in a child's school life and, if absent, developed successfully with focused instruction.

Vast differences exist in the mathematical knowledge of students when they begin the first year of school (R. Wright, 1994). Students who are among the least advanced in their classes tend to remain so throughout their schooling and often give up on mathematics (Aubrey, 1993; R. Wright, 1991, 1994; Young-Loveridge, 1989, 1991). Low attaining students begin to develop strong negative attitudes toward school and mathematics. A sound intervention program has the potential to make a vast difference for many of these students. It can provide them an extended period in which they are successful in number learning, and remove the effects of very early experience of failing at learning, and in particular, failing at math learning. Students' achievement in mathematics is a real gauge to success in K-12 education as those who do not attain the upper levels of mathematics (e.g. Algebra) are often relegated to lower level classes and are denied access to four-year colleges. As the job market continues to see a decline in manufacturing jobs and an increase in service industries, the need for numeracy skills is more evident.

Research has demonstrated that children grow in mathematical skill along a continuum similar to literacy. If we can find where children are tangled along this continuum, we can intervene with deliberate instructional strategies that untangle them and lead to strong mathematical sense. Early intervention programs allow teachers to assess progress along the continuum and provide very specific instructional interventions in order to advance the student to a level at which they can be successful in a regular class.

This proposal describes a proven and reliable program of intervention in early number learning (DfES, 2004, 2005) titled Math Recovery<sup>®</sup>. It was developed in 1992 in response to the problem of chronic failure of too many students in school mathematics and is focused on the second year of school, before "at risk" students are too far behind their class average and have too much experience in failure.

## ***About US Math Recovery<sup>®</sup>***

The US Math Recovery<sup>®</sup> Council is a 501(c)(3) non-profit organization dedicated since 2003 to overseeing Math Recovery<sup>®</sup> programs and its materials, methodology and related intellectual property in the United States. It is the mission of US Math Recovery<sup>®</sup> to transform numeracy education; to connect research with practice; and to empower educators to advance student mathematical thinking and success through Math recovery<sup>®</sup> principles. The vision of US Math Recovery<sup>®</sup> Council is to create and foster attitudes, cultures, and climates that lead to change in mathematics learning and teaching practices. US Math Recovery<sup>®</sup> strives to

- Ensure Math Recovery<sup>®</sup> programs and principles are widely attainable
- Deliver exceptional professional development

- Develop dynamic research-based materials
- Promote program fidelity and professional integrity within the Math Recovery® community
- Contribute to mathematics education research
- Serve as a catalyst for high quality systemic mathematics assessment and instruction
- Empower all students to reach their full mathematical potential.

Math Recovery® is internationally recognized as a highly successful program of intervention in early number learning. The program is used extensively by school systems in the United States, United Kingdom, Ireland and Australia as well as a presence in several other countries such as Canada and Vietnam. Two foundational books provide much of the detail of the one-on-one intervention program (Wright et al., 2000; Wright et al., 2002), and two additional books describe assessment and implications for classroom instruction with 4 – 11 year-olds (Wright et al., 2005; Wright et al., 2012).

Implementation of the program in the United States began in South Carolina in 1995. By 1999 the program had spread to several states in the western region. By 2004 implementation initiatives included school districts in 19 states including New England, California and the mid-west serving approximately 325 teachers, 60 Leaders, and 3000 plus participating students. As of 2014, Math Recovery® has expanded to 36 states and grown to serve 130 Leaders and well over 200,000 students. Additional information about the work of the US Math Recovery® Council can be found at [www.mathrecovery.org](http://www.mathrecovery.org).

### ***Goals and Objectives of the Math Recovery® Program***

The overarching objective for Math Recovery® is to provide a robust intervention framework for teachers working with elementary students to help in the construction of numeracy skills, through assessment, which incorporates a strong analysis component and individualized teaching.

Specifically Math Recovery® goals are to:

- Identify students “at risk” by means of a structured and objective assessment system that allows educators to know exactly where students are in their mathematical development and apply early, short term intervention.
- Provide trained Math Recovery® specialist teachers with the intervention and detailed instructional system that increases student performance in basic arithmetic skill through intensive, individualized instruction.
- Build student confidence as Math Recovery® students work one-on-one with the trained Math Recovery® teacher and help students find success in mathematics and other curriculum areas.
- Offer comprehensive, sustainable and innovative staff development that ensures quality instruction by highly trained teachers.

- Influence and inform curriculum development and school wide mathematics programs, and ensure compatibility with all standards-based math programs.
- Apply a research-based approach to developing numeracy competence giving teachers extensive, current knowledge of assessment and teaching, which is otherwise unavailable.

### ***Features of the Math Recovery<sup>®</sup> Program***

The approach in Math Recovery<sup>®</sup> is first to figure out in detail the student's current knowledge and strategies. Assessments that generate a score or a list of items the student can't answer are not sufficient. Math Recovery<sup>®</sup> teaching sessions are intended to be intensive for the student, and to continually operate at the cutting-edge of the student's current knowledge. For the vast majority of Math Recovery<sup>®</sup> students, focusing and thinking very hard in the teaching sessions and succeeding, is a very positive experience.

The Math Recovery<sup>®</sup> Program has been developed as a systemic response to the problem of chronic failure in school mathematics. The Program encompasses:

- Assessment for identification of the lowest-attaining students at the first-grade level, i.e. the second year of school.
- A Framework for intensive, individualized teaching to these students in order to advance them to a level at which they are likely to learn successfully in a regular class. Math Recovery<sup>®</sup> students undergo a teaching cycle of 12 to 15 weeks in duration. In these teaching cycles students are taught one-on-one 30 minutes daily, for four or five days per week.
- Instructional activities founded on a strong underpinning theory of young children's mathematical learning using a specially developed instructional approach and distinctive assessment procedures.
- A powerful staff development model that builds communities of practice for teachers, leader / trainers and coaching teams.
- A two-year teacher certification program and a one-year Leader/Trainer certification. Math Recovery<sup>®</sup> Teacher/Leader recertification requirements ensure that latest research on the teaching and learning of mathematics is made available.
- An implementation model that is well tested, replicable and is designed to build sustainability at the district level.

### **Assessment**

Assessment in Math Recovery<sup>®</sup> involves the use of assessment schedules in conjunction with the Learning Framework in Number (LFIN). The term 'schedule' is used in the sense of a series of assessment tasks that are presented to the child in a 1-1 interview. The approach to the assessment involves videotaping the assessment interview for subsequent analysis. The assessment has two broad purposes. First, it should provide a rich, detailed description of the child's current knowledge of early number. This rich picture should be in terms of aspects of LFIN. Second, Math Recovery<sup>®</sup> assessment should lead to determination of levels on the relevant tables provided within the LFIN. Other points to note are: (a) Math Recovery<sup>®</sup> assessment does not result in a score; (b) Math Recovery<sup>®</sup> assessment is often used in conjunction with a screening assessment — typically a written assessment of short duration, administered prior to the interview-based assessment.

Add+VantageMR<sup>®</sup> assessments offer tasks that can be used for the screening assessment.

Assessment in the Math Recovery<sup>®</sup> Program draws on extensive programs of research undertaken by Professor Leslie Steffe at the University of Georgia since the late 1970s and by Professor Bob Wright since the early 1980s. Math Recovery<sup>®</sup> teachers are continually assessing students' progress during the teaching sessions through careful observation and review of videotaped teaching sessions. Math Recovery<sup>®</sup> assessment and teaching are fully integrated. Thus assessment informs teaching and teaching provides additional assessment information. Math Recovery<sup>®</sup> assessment accords with a profiles-based approach to assessment with the advantage that it provides much more detailed assessment information than that typically described by profiles. Math Recovery<sup>®</sup> assessment consists of 6 interview-based assessment schedules. Assessment 1.1 and 2.1 are given to determine an initial student profile used to determine initial instruction. Assessment 1.2, 2.2, 3.1 and 3.2 complete the student profile once students are ready for more sophisticated relational concepts.

Assessment 1.1 addresses the following topics:

- Relative sophistication of the child's strategies for adding and subtracting;
- Facility with number words and number word sequences both forward and backward;
- Ability to name numerals;
- Ability to sequence numerals;

Assessment 2.1 addresses the following topics:

- Combining and partitioning numbers to 10;
- Spatial patterns and subitizing, i.e. to correctly ascribe number to spatial patterns when displayed briefly;
- Finger patterns and Base-five strategies;

Assessment 1.2 addresses the following topics:

- Relative sophistication of the child's knowledge of the tens and ones structure of the numeration system;
- Relative sophistication and range of strategies other than counting-by-ones to solve addition and subtraction tasks.

Assessment 2.2 addresses the following topics:

- Advanced grouping: combining and partitioning numbers to 20
- Part-whole construction of numbers
- Relational thinking required for flexible mental computation and the ability to apply mathematics in novel contexts

### **Math Recovery<sup>®</sup> Teaching: The Instructional Framework in Early Number (IFEN)**

In the early years of Math Recovery<sup>®</sup>, instruction involved taking account of the child's current knowledge in terms of the LFIN, and drawing from an extensive bank of instructional settings and activities. In recent years, these instructional settings and activities have been incorporated into an instructional framework for early number (IFEN)

which sets out a progression of key teaching topics which are organized into three strands — number words and numerals, counting and grouping. It includes 30 key teaching topics and more than 180 instructional procedures. Each instructional procedure includes notes on purpose, teaching and students' responses (Robert J. Wright et al., 2006).

For each participant in Math Recovery<sup>®</sup>, the assessment profile is used to develop an individualized teaching framework. Developing the teaching framework involves selecting from an extensive range of instructional settings and teaching activities, which have been especially developed for Math Recovery<sup>®</sup>. Teachers continually endeavor to understand the students' current strategies, and to adjust settings and activities accordingly. In this way the participants are continually undertaking new learning activities, which have the purpose of further extending their knowledge. At the same time activities are selected so that the student has a high likelihood of success and thus is not overwhelmed by a sense of failure. Math Recovery<sup>®</sup> teaching sessions emphasize on-going assessment through careful observation, hypothesizing about the student's current knowledge and strategies, and selecting learning activities closely attuned to the child's current reasoning and strategies. In this way Math Recovery<sup>®</sup> teaching is always at the cutting edge of a student's knowledge.

### **Instructional Activities**

A comprehensive range of instructional activities has been developed for use in the Math Recovery<sup>®</sup> Program. These are comprehensive in the sense that they address what we would regard as all of the important aspects of students' early arithmetical learning. An extensive range of instructional materials is used in these activities. Careful planning and reviewing of videotaped teaching sessions are emphasized. This ensures that the teacher is continually taking full account of the progress being made and adapting the teaching plan in accordance with the child's progress. Evaluation and further development of the instructional activities have occurred in each year of the project since 1992.

### **Instructional Approach**

The instructional approach in Math Recovery<sup>®</sup> is problem-based and from a constructivist perspective. Math Recovery<sup>®</sup> problems are carefully selected to be at a level which will extend the child's current thinking. Thus the student has a good possibility of success but at the same time will find the problem quite challenging. Emphasis is placed on students reflecting on their own mathematical thinking. Direct and intensive teaching is a key feature of the instructional approach. Teaching sessions are routinely videotaped. These videotaped records are extremely useful in the review of teaching and for making appropriate revisions of the teaching program for the child. These reviews and revisions are completed each week of the teaching cycle.

### ***Professional Development for Teachers***

The Math Recovery<sup>®</sup> Program includes new and innovative approaches to teacher professional development. These include routinely using videotaping of assessment and teaching sessions. Excerpts of these videos are used extensively in teachers' meetings and constitute a rich context for discussion and examination of professional practice. Math Recovery<sup>®</sup> has developed a body of theory and practice that is elaborated, research-based, and distinctive. Learning Math Recovery<sup>®</sup> is a long and interesting journey, and for the vast majority of participating teachers, it

seems to be a very enjoyable journey. In addition all a teacher learns in Math Recovery<sup>®</sup> is 100% applicable to classroom teaching and working with the complete range of students.

Candidates for the two-year certification program must be sponsored by their school system. Teachers must have a Bachelor's Degree and at least two years teaching experience at the primary level with at least one year of primary math. Additional skills required include: good communication skills and the desire to work with other teachers, parents, and administrators; ability to organize and keep accurate student records; a personal commitment to self-reflection and learning; and, willingness to make a two-year commitment to participate as a teacher in Math Recovery<sup>®</sup>.

The initial phase of the course involves classes totaling 60 hours of instruction and covers a detailed introduction to the theory, techniques and assessment of the Math Recovery<sup>®</sup> Program. The course may be offered as a single block or divided into two or three segments – one full-time week and monthly half-day seminars. After the initial phase of the course the Math Recovery<sup>®</sup> teacher typically would spend the morning part of each working day teaching four thirty-minute, individualized teaching sessions. This phase begins with the assessment of potential participants, selection of an initial group of four participants and development of initial teaching frameworks allowing teachers to begin working with students immediately.

The US Math Recovery<sup>®</sup> Council recognizes certification as a Math Recovery<sup>®</sup> Intervention Specialist. The candidate must meet the following criteria: successfully complete Math Recovery<sup>®</sup> initial 60-hour training; attend and participate in all continuing contact sessions; complete a minimum of 100 hours of one-on-one, videotaped instruction; and successfully complete and submit assessment and case study projects for review. Every three years Math Recovery<sup>®</sup> Intervention Specialists need to complete programs requirements to maintain a current Math Recovery<sup>®</sup> Intervention Specialist certification.

Leader training is a one-year program that begins with a six-day intensive training with program developers. Leaders are able to teach up to 8 teachers in their first year of practice and 15 teachers once they have been certified. They are responsible for coaching, oversight and support of a cadre of teachers. In addition, they periodically review teacher assessments and written records to ensure reliability.

Developing a strong Leaders' cohort is the corner stone of Math Recovery<sup>®</sup> implementation – the return on investment is many-fold. An experienced Leader can train as many as 15 teachers a year and mentor 6 as part of ongoing teacher development. If each teacher can teach ten to twelve of the lowest-attaining first graders each school year, conceivably a single leader could result in positive intervention for nearly 800 students over a five year period.

## *Theoretical Basis of Math Recovery<sup>®</sup> and Current Research*

Mathematics Recovery (Phillips, Leonard, Horton, Wright, & Stafford, 2003; R. Wright, 2000, 2003; R.J. Wright, Stanger, Stafford, & Martland, 2006) is grounded in over forty years of mathematics education research. The Mathematics Recovery Program was the outcome of a three-year research and development project at Southern Cross University in northern New South Wales, conducted from 1992-1995. The project received major funding from the Australian Research Council in conjunction with major contributions in the form of teacher time from the regional government and Catholic school systems. Over the course of the project, 20 teachers and over 200 students participated in the development of interview schedules, principles for a general approach to teaching, the Learning Framework in Number (LFIN), and instructional settings and procedures. In Mathematics Recovery, the Learning Framework in Number (LFIN) is regarded as a rich description of the children's early number knowledge. Alternatively, one could say children's early arithmetical knowledge. The term 'a child's early number/ arithmetical knowledge' is used in the sense of everything that the child can be said to know about early number. It is important to see these aspects as inter-related rather than distinct. The LFIN spans eleven essential aspects of children's early numerical knowledge and provides essential guidance for assessment and teaching in early number. These aspects include Stages of Early Arithmetical Learning (SEAL) adapted from research by Steffe and colleagues (L. Steffe, Glasersfeld, Richards, & Cobb, 1983; Leslie P. Steffe, 1992; Leslie P Steffe, Cobb, von Glasersfeld, & Sinclair, 1988) and related research by Wright (1989; 1991a); the development of base-ten arithmetical strategies (Cobb & Wheatley, 1988); forward and backward number word sequences (R. Wright, 1991, 1994); combining and partitioning numbers (Cobb, Wood, & Yackel, 1991, 1992; Yackel, Cobb, & Wood, 1991); spatial patterns and subitizing (Glasersfeld, 1982); temporal sequences (Wright1, 1989); Finger Patterns; Five-based strategies (Cobb, McClain, Whitenack, & Estes, 1995; Gravemeijer, 1994; Gravemeijer, Cobb, Bowers, & Whitenack, 2000); and Multiplication and Division (Mulligan & Mitchelmore, 1997).

Steffe and colleagues conducted longitudinal teaching experiments with 1<sup>st</sup> and 2<sup>nd</sup> graders, focusing on early number. Typically these extended over a two-year period and involved several teaching cycles — periods of up to 18 weeks, during which participants were taught individually in 30 minute sessions, twice weekly. The goal of this research was to develop psychological models of conceptual development over extended periods of time. Steffe's approach was paradigmatically distinctive in terms of its research design (i.e. the teaching experiments just described) and techniques, and also in terms of its theoretical orientation. Distinctive aspects of the research techniques include an intensive, problem-based approach to instruction and routinely videotaping assessment and teaching sessions for retrospective analysis. Steffe's theoretical orientation was Piagetian and drew on von Glasersfeld's (1995) theory of cognitive constructivism, which in turn draws extensively on Piaget's theory of cognitive development. The research base of Mathematics Recovery also includes Wright's doctoral research — a teaching experiment relating to numerical development in the Kindergarten year (Wright, 1989), research into the numerical knowledge of school entrants (Wright, 1991) and the progression of number learning in the first two years of school (Wright, 1994).



Because of the dynamic nature of the program, the latest research is continually incorporated into the theory and practice of Math Recovery<sup>®</sup> and Add+VantageMR<sup>®</sup> assessment, instruction and professional development. Wright et al (2012) extends this early work to include assessment, teaching and intervention focused at 7-11 year-olds.

Math Recovery<sup>®</sup> can be regarded as consisting of two distinct parts. One part concerns an elaborated body of theory and techniques for working with children, that is, teaching early number knowledge. The second concerns innovative ways of working with teachers to provide ongoing, effective professional development and coaching in order to learn about working with children (Bobis et al., 2005).

MacLean (2003) evaluated the relative effectiveness of three different professional development models on low-achieving, urban first graders. The first model consisted of a full Math Recovery<sup>®</sup> implementation. The full implementation included intensive one-on-one intervention provided to selected, low-achieving, Title 1 first grade children as well as on-going professional development for classroom teachers provided by the on-site Math Recovery<sup>®</sup> leader. This professional development took the form of presentations, joint planning sessions, demonstration and team teaching. The second model involved those same Math Recovery<sup>®</sup> leaders conducting on-going professional development in Math Recovery<sup>®</sup> theory, strategies, and activities to classroom teachers from schools without an on-site, one-on-one intervention component. These strategies and activities were adapted for use in the classroom setting. The third model involved schools in which classroom teachers received periodic, “one-shot” professional development and conference attendance. This professional development was provided by both in-district math leaders and by outside consultants and speakers. The teachers from this model were not exposed to any of the Math Recovery<sup>®</sup> theory and methods. MacLean found that the full Math Recovery<sup>®</sup> implementation model significantly out-performed both the on-going professional development only model as well as the periodic, “one-shot” model. MacLean’s findings are similar to other researchers (Phillips et al., 2003). Williams (2001) found that Math Recovery<sup>®</sup> significantly changes teacher practice in the classroom. Teachers participating in the Math Recovery<sup>®</sup> training became much more reform oriented in their teaching.

### ***Math Recovery<sup>®</sup> Programs and Special Education (I.D.E.A.)***

The federal guidelines for special education mandate that interventions be documented prior to a child being placed in special education. Mathematics Recovery provides an intensive intervention specifically geared to each child’s individual needs. In many cases, what may appear to be a learning disability in mathematics is in fact a lack of experience with mathematics. Mathematics Recovery provides a conduit for that experience.

Furthermore, Math Recovery<sup>®</sup> can be a powerful vehicle for insuring that special educators are highly qualified. Mathematics Recovery teacher training equips teachers with knowledge of the developmental Learning Framework in Number. Within this framework, Mathematics Recovery provides a diagnostic interview that affords teachers an accurate profile of the child’s mathematical understanding so that an individualized education plan can address specific needs. The Mathematics Recovery Teaching Framework in Number enables special educators to properly scaffold instruction so that learning can be accelerated for maximum growth.

### ***Additional Benefits of Math Recovery® Programs***

Math Recovery® teachers consistently report they have learned new ways to understand young children's mathematical thinking and new methods of teaching mathematics. Additionally, in many of the participating schools, the project has resulted in important changes to classroom teaching methods in K-2 mathematics. The theory and techniques learned by the participating teachers are equally applicable to average or able learners as well as low-attaining students. Particularly gratifying is that students' participation in the Math Recovery® program typically is accompanied by dramatic improvements in self-esteem, classroom behavior and attitude towards learning. Classroom teachers in the participating schools report increased efficacy in teaching mathematics due to the informative nature of assessments and how to use assessment data when making instructional decisions. Teachers leave with an understanding of how children learn initial mathematics concepts and the strategies children use when solving problems. Because of its success to date Math Recovery® is gaining increasingly strong support from parents, teachers, principals and senior administrators.

### ***Professional Growth***

The US Math Recovery® Council sponsors an annual conference to provide the latest domestic and international research. This is an opportunity to work closely with Math Recovery® program developers as well learn from fellow colleagues from across the Nation. Each year, US Math Recovery® also sponsors a Summer Institute opportunity where individuals learn to facilitate courses and increase their own Math Recovery® knowledge from US Math Recovery® developers and leaders.

### ***Validity and Reliability Measures***

Smith, Cobb, Farran, Cordray, and Munter (2013) determined Math Recovery® assessments to have high construct validity; indicating that Math Recovery® assessments indeed measure student mathematical understanding as described in the Learning Framework in Number. Munn (2007) found Math Recovery® assessment to have a 74% agreement when corrected for chance, which is a very respectable interrater reliability given the complex coding scheme necessary to capture children's early numeracy strategy development.

### ***Math Recovery® Results***

A 2-year experimental design research evaluation of Math Recovery® was conducted from 2007-2009 in 20 elementary school across five school districts in two states (Smith et al., 2013). In all, 343 children received one-on-one Math Recovery® intervention over the two years. During year one 72.5% of the participants received free and reduced lunch, 46.3% were non-white and 16.3% were limited English proficiency. In year 2, 55.6% received free and reduced lunch, 46.5% were non-white and 8.8% were limited English proficiency. Despite just 10% of participating students receiving the recommended number of 45-60 Math Recovery® lessons, there was a positive causal effect of Math Recovery® intervention at the end of first grade. As diagnostic program, effect sizes were found to be between +0.30 and +0.40 for students participating in Math Recovery® intervention.

The longitudinal effects of Mathematics Recovery substantiate the value of the program. Mathematics Recovery was fully implemented during the 1999-2000 school year at Roye-

Williams Elementary School (RWES), a Title 1 school in Harford County, Maryland. One Math Recovery<sup>®</sup> teacher plus three paraprofessionals were dedicated full-time to mathematics instruction and professional development. Prior to the implementation of Math Recovery<sup>®</sup>, only 30-40% of children scored proficient or better in mathematics on the Maryland state assessment program (MSPAP). In 2004, the fifth year of implementation, 76.6 % of fifth grade children were proficient or better in mathematics on the Maryland State Assessment (MSA); 82.9% of fourth grade children scored proficient or better with 22.9 % scoring advanced; and 72.8 % of third grade children scored proficient or better.

Category (5 <sup>th</sup> Grade)	RWES	Harford Co.	Maryland
Advanced	10.4 %	12.4 %	12.7 %
Proficient	66.2 %	62.1 %	50.4 %
Basic (Did not meet standard)	23.4 %	25.5 %	36.9 %

*No Child Left Behind* Disaggregated Data for 2004 5th Graders

Category	% of Children Proficient or Better		
	RWES	Harford Co.	Maryland
Africa American	75.0%	54.8 %	45.9%
White	75.7%	78.5%	76.3%
Hispanic	85.7%	70.3%	52.1%
Limited English Proficient	76.3%	65.0%	36.0%
504 Services	76.6%	81.0%	59.1%
Free & Reduced Lunch	60.7%	56.1%	44.4%
Special Education	41.7 %	40.8%	29.7%

Category (4 <sup>th</sup> Grade)	RWES	Harford Co.	Maryland
Advanced	22.9 %	20.3 %	20.0 %
Proficient	60.0 %	58.8 %	49.6 %
Basic (Did not meet standard)	17.1 %	20.9 %	30.4 %

*No Child Left Behind* Disaggregated Data for 2004 4th Graders

Category	% of Children Proficient or Better		
	RWES	Harford Co.	Maryland
Africa American	77.5 %	57.9 %	51.9 %

White	83.3 %	84.0 %	83.2 %
Hispanic	91.7 %	73.7 %	59.3 %
Limited English Proficient	NA	45.0 %	38.8 %
504 Services	NA	68.7 %	63.9 %
Free & Reduced Lunch	76.9 %	61.1 %	51.4 %
Special Education	62.5 %	48.5 %	38.5 %

Category (3 <sup>rd</sup> Grade)	RWES	Harford Co.	Maryland
Advanced	7.4 %	22.1 %	19.9 %
Proficient	65.4 %	59.0 %	52.3 %
Basic (Did not meet standard)	27.2 %	19.0 %	27.8 %

<i>No Child Left Behind</i> Disaggregated Data for 2004 3 <sup>rd</sup> Graders			
Category	% of Children Proficient or Better		
	RWES	Harford Co.	Maryland
Africa American	75.9 %	67.1 %	51.3 %
White	76.3 %	84.0 %	64.5 %
Hispanic	50.0 %	68.7 %	58.2 %
Limited English Proficient	NA	50.0 %	49.9 %
504 Services	NA	100 %	62.6 %
Free & Reduced Lunch	75.0 %	65.3 %	55.9 %
Special Education	44.4 %	51.1 %	42.1 %

In 2006, the Kentucky Center for Mathematics was formed through an appropriation given to Northern Kentucky University with the goal of enhancing the mathematics within the state of Kentucky. This statewide project includes both Math Recovery<sup>®</sup> and AVMR and continues today. Math Recovery<sup>®</sup> is one of two intervention programs implemented as a part of the Primary Mathematics Intervention Program. Initial program results indicated that students participating in Math Recovery<sup>®</sup> Intervention gained slightly more than 2 grade levels and 70% of participants attained expected grade level proficiency after participating in Math Recovery<sup>®</sup> Intervention. Results not only identified significant student achievement gains, but also documented change in teachers' beliefs, attitudes and content knowledge towards mathematics. Math Intervention Teachers (MITs) in Math Recovery<sup>®</sup> Intervention reported an increase in

mathematics content knowledge, perceived themselves to better identify best practices for classroom instruction that support teaching reasoning and problem solving skills to students, and moved away from the believe that mathematics is mainly using rules. MITs also reported greater knowledge of Kentucky core content standards (50% rate of agreement prior to participating to 100% at end of year 1). Results from a 2011-2012 external evaluation of the Primary Mathematics Intervention Program at KCM indicated that the program enables sustained teacher growth, yields sustained gains in intervention students' achievement, encourages school-wide changes toward effective instructional practices and enhances school-wide gains in student achievement. In 2014, Math Recovery<sup>®</sup> Intervention and Add+VantageMR<sup>®</sup> programs are 2 of the five approved intervention programs by the Kentucky Department of Education that elementary schools can implement with Kentucky Mathematics Achievement Funds. More information about Kentucky's implementation can be found on the Kentucky Center for Math website, [www.kentuckymathematics.org](http://www.kentuckymathematics.org).

### ***Math Recovery<sup>®</sup> Implementation Models***

Math Recovery<sup>®</sup> is a flexible program that can be tailored to fit the needs of the local school district. The long-term success of the program is built upon the sustained commitment to professional development and the added benefit of Math Recovery<sup>®</sup> Intervention Specialists serving as a resource for Add+VantageMR<sup>®</sup> educators. Math Recovery<sup>®</sup> programs are implemented in a variety of ways. Districts may choose to establish elementary mathematics teacher specialists or they may choose to establish intervention specialists who support one-on-one and small group mathematics intervention. Districts have also chosen to establish Math Recovery<sup>®</sup> Intervention expertise with special education teachers and then provide Add+VantageMR<sup>®</sup> expertise for classroom teachers to establish a common mathematics experience for students and a common language for educators within a building and district. As educators become more in-tuned to student thinking, they begin to analyze the stages and levels of all of their math students, and they begin differentiating instruction based on informal assessment from observations during the regular math classroom instruction.

#### **Models Involving a Full or Half Time Math Recovery<sup>®</sup> Specialist**

In this model, a Math Recovery<sup>®</sup> Intervention Specialist completes a number of one-on-one cycles each semester. This individual might also provide on-going professional development for the classroom and special education teachers in that school. This professional development might take the form of demonstration lessons, team teaching, team planning, mentoring, facilitating study groups, presentations at faculty meetings, etc. One disadvantage to this model is that it is not perceived as being cost effective for a teacher to only be able to teach 5 to 6 students each semester; but it is the only delivery system that has proven to be effective for some at-risk students. Over the course of the school year a full-time Math Recovery<sup>®</sup> Intervention Specialist would be expected to teach ten to twelve of the lowest -attaining first- and second-graders in the school.

#### **Models Involving a Full Time Math Recovery<sup>®</sup> Specialist with Small Groups**

In this variation, the Math Recovery<sup>®</sup> Intervention Specialist would work with small groups of children (up to 3 children in group) in addition to one-on-one cycles. This allows the specialist to teach more children; however, small-group instruction is not as effective as one-on-one instruction. Children not adequately progressing in small groups may benefit the most from one-

on-one instruction.

## ***Math Recovery<sup>®</sup> Teacher Roles and Responsibilities***

### **Certification Requirement**

- Participate in a 60-hour professional development course that includes theory of early childhood mathematics and the Math Recovery<sup>®</sup> assessment and teaching techniques.
- Attend all class sessions and complete assignments and readings as required.
- Demonstrate proficiency in teaching techniques with a Math Recovery<sup>®</sup> student, sharing videotaped excerpts in training sessions and collegial meetings.
- Complete a minimum of 100 hours of one-on-one, videotaped instruction
- Successfully complete assessment and case study projects
- Obtain Math Recovery<sup>®</sup> Leader recommendation for Math Recovery<sup>®</sup> Specialist Teacher certification to US Math Recovery<sup>®</sup> Council

### **On-going Requirements**

- Administer assessments to students at the start of the school year to identify those most in need of Math Recovery<sup>®</sup> instruction.
- Spend part of the workday teaching Math Recovery<sup>®</sup> students individually for 30 minutes daily, videotaping each lesson for planning and reflection.
- Keep detailed and accurate records on each student – daily lesson plans, attendance, journal entries, etc.
- Make recommendations for the discontinuation of students from the program.
- Monitor progress of students discontinued from the program. Observe students during classroom mathematics instruction to ensure transfer.
- Administer pre and post-assessments and other required tests to current, discontinued, and comparison students according to established guidelines for evaluation. These are to be submitted on time according to the schedule for US Math Recovery<sup>®</sup> data collection.
- Secure permission from parent(s) for the student to be videotaped during lessons and assessments.
- Communicate with first grade teachers of Math Recovery<sup>®</sup> students through regular conferences.
- Communicate with school personnel, including the principal, on the progress of the Math Recovery<sup>®</sup> Program.
- Attend the monthly group continued learning meetings and end-of year report meeting.
- If possible, receive and make two observation visits to other Math Recovery<sup>®</sup> teachers each year.

### **Professional Development Opportunities**

- Encourage first grade teachers to observe a lesson and respond to questions regarding Math Recovery<sup>®</sup> techniques.
- Serve as a faculty resource for mathematics strategies.
- Receive visits and discuss program with visitors from other sites.

Math Recovery<sup>®</sup> Teacher candidates successfully completing the professional development program receive Math Recovery<sup>®</sup> Specialist Teacher certification issued by the US Math

Recovery<sup>®</sup> Council. The candidate must meet the certification requirements listed above.

***Related Programs Throughout the World:*** *Math Recovery<sup>®</sup>, Count Me In Too, Count Me In Too Indigenous, Numeracy Development Project, Number Framework, Counting On, Add+VantageMR<sup>®</sup> (AVMR), Student Numeracy Assessment Progressions (SNAP), (Melbourne Project)*

Math Recovery<sup>®</sup> Early Intervention and the Learning Framework in Number have been the basis for numeracy projects and frameworks in Australia, the United States, United Kingdom, and Canada. Math Recovery<sup>®</sup> Early Intervention and the Learning Framework in Number were first used in schools in New South Wales, Australia, as part of a grant funded by the regional government and Catholic school systems from 1992 to 1995. The success of this intervention program was followed by the expansion of Math Recovery<sup>®</sup> to the United States and the United Kingdom, including Ireland, Scotland, Wales and England. The Department of Education and Teaching for New South Wales, Australia produced classroom applications of Math Recovery<sup>®</sup>, *Count Me in Too* and *Count Me in Too Indigenous*. The Count Me in Too programs have undergone evaluation since 2000 (see [http://www.curriculumsupport.education.nsw.gov.au/primary/mathematics/countmeintoo/evaluation\\_rep.htm](http://www.curriculumsupport.education.nsw.gov.au/primary/mathematics/countmeintoo/evaluation_rep.htm) and [http://www.curriculumsupport.education.nsw.gov.au/primary/mathematics/countmeintoo/indigenous/indig\\_evaluations.htm](http://www.curriculumsupport.education.nsw.gov.au/primary/mathematics/countmeintoo/indigenous/indig_evaluations.htm)).

Further application of Dr. Wright's Learning Framework in Number and the pedagogy and pedagogical tools within Math Recovery<sup>®</sup> have influenced New Zealand's Numeracy Development Projects and resulting Number Framework (see <http://www.nzmaths.co.nz/nz-number-framework-and-teaching-model>). The New South Wales Department of Education extended the application to 11-13 year old students through the development of *Counting On*. Since 2006, *Add+VantageMR<sup>®</sup> (AVMR)*, a classroom adaptation of Math Recovery<sup>®</sup> Early Intervention, has been incorporated into district professional development and assessment plans. AVMR as developed by Dr. Wright, Kurt Kinsey, Lucinda "Petey" MacCarty, Dr. Pam Tabor, and Amy Earnst is a way to bring Math Recovery<sup>®</sup> Intervention theory, assessment and instruction to classroom teachers. AVMR is used in schools throughout the United States.

Since 2004, Dr. Wright has been furthering the research surrounding the implementation of the Learning Framework in Number through the Numeracy Intervention Research Project funded by the Australian Research Council, the Catholic Education Office Melbourne and the Victorian Catholic Education Commission. This project focused on developing pedagogical tools for intervention targeting numeracy at third and fourth grade (Robert J. Wright, Ellemor-Collins, & Tabor, 2012).

Dr. Wright has been involved in the development and implementation of each of these projects across the world and continues to be an active contributor to the work internationally as well as in the United States.

***Goals and Objectives of Add+VantageMR<sup>®</sup> Program***  
**Professional Development that Integrates with Your Curriculum**

- Provide a unique approach to professional development for improving elementary numeracy instruction and student achievement.
- Support K-5 mathematics assessment and teaching through innovative approaches to understanding children's mathematical thinking.
- Provide teachers with resources to analyze and modify instructional tasks to meet the needs of a diverse range of students. This expertise allows teachers to customize any commercial mathematics package specifically for the individual children in their classroom.

Add+VantageMR<sup>®</sup> is not a curriculum. Instead, teachers use any curriculum package more effectively.

### Features

- A detailed understanding of how children develop early numeracy via the Learning Framework for Early Number. (Wright, 2006)
- Efficient and effective assessment tools to recognize their students' current understandings of number concepts, and to support data-driven instruction. Collegial teams developed during the course provide for ongoing study.
- Videotaped interview-based assessment (Ellimore-Collins and Wright, 2008) that utilizes informed dynamic assessment ideal for reluctant learners and underachievers, as well as highly competent students.
- The diagnostic nature of the assessments and the classroom profiles improve data management and allow for more systematic differentiated instruction.
- Course participants practice administering and analyzing their videotaped assessments in collegial teams with the support of the Add+VantageMR<sup>®</sup> course facilitator.

### Add+VantageMR<sup>®</sup> Course 1

Provides four days of intensive professional development and covers:

- Addition & Subtraction
- Number Sequences
- Numeral Identification
- Structuring Numbers

Course 1 participant materials include:

- The Add+VantageMR<sup>®</sup> teacher handbook
- Three assessment kits, exemplary teaching settings
- *Teaching Number in the Classroom with 4-8 Year Olds* by Robert Wright, et al.
- This four-day course has no pre-requisites and is appropriate for classroom teachers, special education teachers, Title 1 teachers, para-professionals, instructional coaches, administrators, and interested parents.

### Add+VantageMR<sup>®</sup> Course 2

Course 2 involves four days of professional development beyond Course 1 that extends the teacher's knowledge of the Learning Framework in Number (LFIN) in the following areas:



- Conceptual Place Value
- Multiplication and Division

Course 2 participant materials include:

- The Add+VantageMR<sup>®</sup> teacher handbook
- Blackline Masters handbook with exemplary teaching activities
- Two assessment kits
- Exemplary teaching settings for Multiplication/Division and Place Value
- *Developing Number Knowledge: Assessment, Teaching & Intervention with 7-11-Year-Olds* by Robert Wright, et al.
- Add+VantageMR<sup>®</sup> Course 1 is a pre-requisite to Course 2 and is appropriate for classroom teachers, special education teachers, Title 1 teachers, para-professionals, instructional coaches, administrators, and interested parents.

Add+VantageMR<sup>®</sup> uses the same approach to assessment and instruction as Math Recovery<sup>®</sup> Intervention. The development of AVMR in 2006 made the Learning Framework in Number accessible to classroom teachers and all educators within a school resulting in a common language and common instruction for students in the classroom and intervention settings. Add+VantageMR<sup>®</sup> provides educators with assessment tasks to flexibly group students according to specific mathematics concepts and learning goals. Using the resulting student profiles, instructional settings are chosen from Math Recovery<sup>®</sup> publications and district adopted curriculum.

### ***Math Recovery<sup>®</sup> Programs and Response to Intervention***

Add+VantageMR<sup>®</sup> and Math Recovery<sup>®</sup> Intervention Specialist professional development provide diagnostic assessment and proven instructional techniques that offer instruction at varying levels of intensity. Add+VantageMR<sup>®</sup> strengthens core instruction at the Tier 1 level and includes assessment tasks for use as screening tools to determine individual students for tier 2 supplemental intervention. Both Add+VantageMR<sup>®</sup> and Math Recovery<sup>®</sup> Specialist intervention include a model of moderate intensity of instruction through small group instruction to meet children's educational needs. Math Recovery<sup>®</sup> and Add+VantageMR<sup>®</sup> small group instruction applies to the secondary prevention area of the RTI framework. Teachers work with groups of 2 or 3 students, 4-5 days per week for a minimum of 10 weeks. A detailed profile is determined for each student and instruction is targeted to the cutting edge of students' knowledge in two to four areas of the Learning Framework in Number. Tertiary, tier 3, prevention involving the most intense instruction is one-on-one Math Recovery<sup>®</sup> Intervention as experienced through the Math Recovery<sup>®</sup> Intervention Specialist program.

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